## Exercise 25

Find the gradient vector field $\nabla f$ of $f$ and sketch it.

$$f(x,y) = \frac{1}{2}(x-y)^2$$

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**Solution**

Calculate the gradient and call it $\mathbf{F}$.

$$\begin{aligned}
\mathbf{F} &= \nabla f \\
&= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\rangle f \\
&= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \\
&= \left\langle \frac{\partial}{\partial x}\left[\frac{1}{2}(x-y)^2\right], \frac{\partial}{\partial y}\left[\frac{1}{2}(x-y)^2\right] \right\rangle \\
&= \left\langle \frac{1}{2}\left[2(x-y)\cdot\frac{\partial}{\partial x}(x-y)\right], \frac{1}{2}\left[2(x-y)\cdot\frac{\partial}{\partial y}(x-y)\right] \right\rangle \\
&= \left\langle \frac{1}{2}\left[2(x-y)\cdot(1)\right], \frac{1}{2}\left[2(x-y)\cdot(-1)\right] \right\rangle \\
&= \langle x-y, y-x \rangle
\end{aligned}$$

The vector field of this gradient is superimposed on a contour plot of $f(x,y)$. Notice that the vectors are perpendicular to each of the contours, pointing in the direction of greatest increase.